

# The Optimum Received Power Levels of Uplink Non-Orthogonal Multiple Access (NOMA) Signals

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**Abstract**— Non-orthogonal multiple access (NOMA) has been recently considered as a promising multiple access technique for fifth generation (5G) mobile networks as an enabling technology to meet the demands of low latency, high reliability, massive connectivity, and high throughput. The two dominants types of NOMA are: power-domain and code-domain. The key feature of power-domain NOMA is to allow different users to share the same time, frequency, and code, but with different power levels. In code-domain NOMA, different spread-spectrum codes are assigned to different users and are then multiplexed over the same time-frequency resources. This paper concentrates on power-domain NOMA. In power-domain NOMA, Successive Interference Cancellation (SIC) is employed at the receiver. In this paper, the optimum received uplink power levels using a SIC detector is determined analytically for any number of transmitters. The optimum uplink received power levels using the SIC decoder in NOMA strongly resembles the  $\mu$ -law encoding used in pulse code modulation (PCM) speech companders.

**Keywords**— Non-orthogonal multiple access (NOMA); 5G; power domain; successive interference cancellation (SIC); orthogonal multiple access (OMA); pulse code modulation (PCM).

## I. INTRODUCTION

The increasing demand of Internet of Things and mobile internet imposes several challenging requirements for 5G wireless communications, e.g., high throughput, low latency, high reliability, and massive connectivity. Generally, one important component that affects the system capacity is the multiple access schemes. Multiple access technology allows multiple users to share the available radio resources in a cost-effective and a spectrum-efficient manner [1]. Frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA) were introduced as multiple access schemes in 1G, 2G, and 3G, respectively. Orthogonal frequency division multiple access (OFDMA) and single-carrier frequency division multiple access (SC-FDMA) were adopted as orthogonal multiple access (OMA) technology in 4G, Long-Term Evolution (LTE) and LTE-Advanced that were standardized by the 3<sup>rd</sup> Generation Partnership Project (3GPP) [2]. However, to meet the requirements for 5G wireless communications systems, new solutions must be provided to respond to the target applications.

To support applications such as the Internet of Things (IoT) [3], a downlink version of NOMA is standardized in the 3GPP LTE Advanced (3GPP-LTE-A) under the name Multi-User Superposition Transmission (MUST) [4].

NOMA has the potential to accommodate massive connectivity and increase the system throughput, allowing multiple users to share the same resources, either in time, frequency, or code via power-domain or code-domain multiplexing techniques [5]-[8]. This paper focuses on the power-domain techniques with the main objective being to determine the optimum power levels at the receiver for any number of transmitters. In power-domain NOMA, SIC is used at the receiver in the uplink and downlink, respectively [9]-[10]. The details of using SIC are presented in Section II. As a comparison between OMA and NOMA, for massive connectivity, NOMA can accommodate more users than OMA since the number of supported users or devices is not limited by the amount of the available resources [11]. The transmission latency and signaling overhead are reduced in NOMA because there is no scheduling in most uplink NOMA schemes, which leads to a grant-free uplink transmission [9]. Furthermore, a key feature of NOMA is that it introduces a balanced trade-off between user fairness and system throughput that is an important feature for 5G when it is used to support the Internet of Things (IoT) applications of high number of devices that are connected with each other, but with limited data utilization. The attractiveness of NOMA is that it is matched to many IoT applications, where fewer channels are needed to serve large numbers of sensors due to limited throughput, or utilization requirements of the IoT devices [12]-[13].

NOMA has some challenges that need to be addressed for widespread adoption in 5G. For hybrid multiple access, the challenge is how to combine NOMA with other multiple access techniques for use in 5G in addition to conventional OMA techniques [6]. In the downlink, the lack of channel state knowledge impacts the system performance, since the channel state information (CSI) for each user must be learned by all users and the base station (BS) allocates power to each user based on its CSI [14]. Due to that issue, it is likely that NOMA will not be practical in the downlink, especially, when the number of users is large as envisioned in IoT networks. An additional challenge that limits the use of NOMA in 5G, for IoT devices, is the trade-off between system performance and receiver complexity [1].

The remainder of this paper is organized as follows. Section II, describes the system model illustrating the concept of applying NOMA to the uplink with ideal SIC reception [16] at the base station (BS) and derives the optimum received power level as a function of the number of signals. Evaluating the derived formula of the optimum received power levels using MATLAB is presented in Section III. Finally, Section IV summarizes the conclusions of the paper.

## II. SYSTEM MODEL

The system model scenario is shown in Fig. 1 for the case of three users, and we assume that SIC operates ideally (that is SIC makes perfect decisions). Since the users are ordered by their signal strength, the SIC detector [16] first decodes the strongest signal and then subtracts it from the combined received signal and then the second strongest signal can be detected and subtracted from the composite signal, and this process continues until all the signals are detected [17]. As discussed in [18], using NOMA with SIC exploits the SINR difference among users because of the non-uniform power allocation at the user transmitters.

The optimum received power level is determined for each signal so as to achieve the same bit error rate (BER) for each received signal. The system model scenario is shown in Fig. 1 for the case of three users. As shown in Fig. 1, the three transmitters (users) signals  $x_1$ ,  $x_2$ , and  $x_3$  have power levels of  $P_1$ ,  $P_2$ , and  $P_3$ , respectively. Assume that  $x_1$  represents the weakest signal and,  $x_3$  represents the strongest signal and the received signal at the BS is modeled as

$$y = h_1 x_1 + h_2 x_2 + h_3 x_3 + n, \quad (1)$$

where  $y$  is the received signal at the base station. The channel coefficients are denoted as  $h_i$  ( $i = 1, 2, 3$ ). The parameter  $n$  is the noise, which is assumed to be Additive White Gaussian Noise (AWGN) with zero mean and variance  $\sigma_n^2$ .

The use of SIC at the base station receiver is shown in Fig. 2. After the channel coefficients  $h_i$  ( $i = 1, 2, 3$ ) are estimated, SIC is used at the BS to decode the three received signals in three stages. As described above, first, the received signal  $x_3$ , which is the strongest signal, is decoded and the other two signals are considered as noise (or interference). This means that the signal  $x_3$  is decoded directly without applying SIC. In the second stage, the decoded signal of  $x_3$  is subtracted from the received signal and the signal  $x_2$  is detected and  $x_1$  is treated as noise. Finally,  $x_1$  is decoded after subtracting the decoded signal of  $x_2$ .

To achieve the same BER for each signal, the optimum received power level for each transmitter is computed and then compared with a specific value (threshold value) of SINR. The optimum received power level is derived for the case of three transmitters and then it is generalized for  $N$  transmitters.

To derive the general formula for the optimum received power levels, the signal power of the first transmitter  $P_1$  is determined and then it is compared with the required threshold SINR value which is given in decibel value (dB).

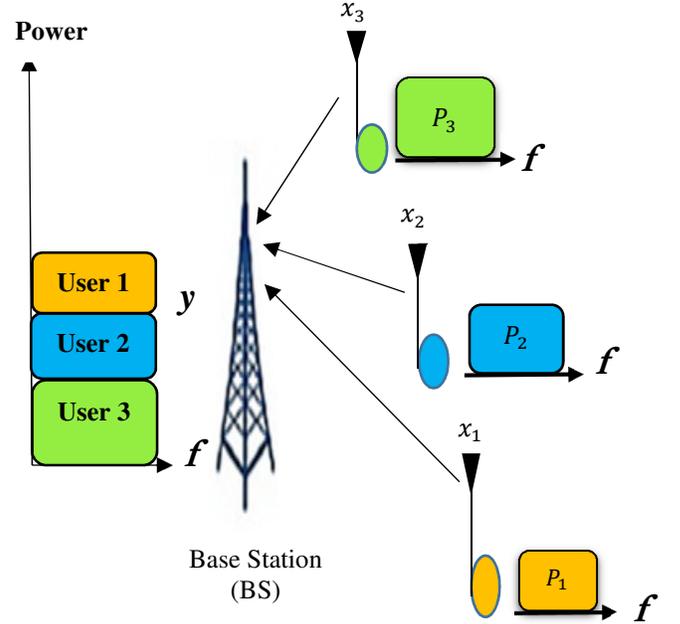


Fig. 1. Uplink power-domain NOMA with ideal SIC reception.

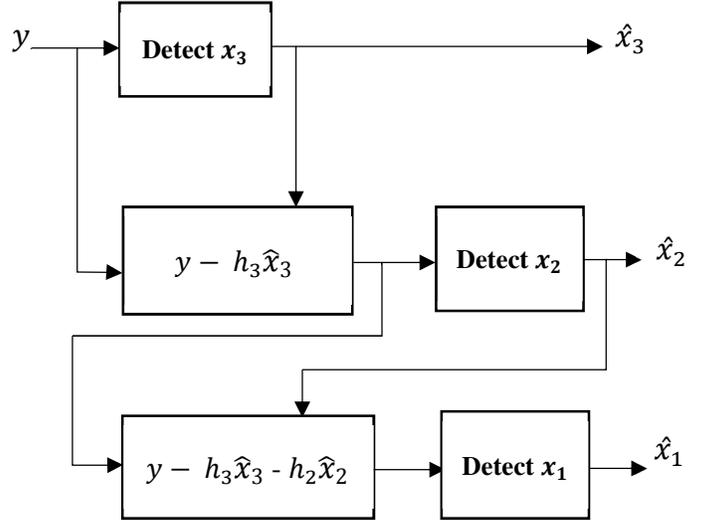


Fig. 2. Illustration of SIC detection of the signals at the base station with three users.

The required SINR is assumed to be the same for each signal. The value  $P_1$  is such that  $x_1$  can be accurately received, that is

$$\frac{P_1 h_1}{\sigma_n^2} = \text{SINR} \quad (2)$$

so that

$$P_1 = \frac{\sigma_n^2}{h_1} \text{SINR}. \quad (3)$$

Similarly, the second transmitter power,  $P_2$ , is computed as

$$P_2 = \frac{\sigma_n^2}{h_2} \text{SINR} (\text{SINR} + 1). \quad (4)$$

As observed in (4), the power value  $P_2$  depends on the power value of the first transmitter  $P_1$ . The same rule is iteratively applied to determine  $P_3$ , which depends on the previous values of  $P_1$  and  $P_2$  and is determined to be

$$P_3 = \frac{\sigma_n^2}{h_3} \text{SINR} (\text{SINR}^2 + 2 \text{SINR} + 1). \quad (5)$$

This iteration may be extended for  $N$  transmitters and the optimum received power level is a function of the noise power value,  $\sigma_n^2$ , channel coefficient  $h_i$ , and the required  $\text{SINR}$  as

$$P_i = \frac{\sigma_n^2}{h_i} \text{SINR} (1 + \text{SINR})^{i-1}, i = 1, 2, \dots, N. \quad (6)$$

As shown in (6), for an AWGN channel and constant  $\text{SINR}$ , the optimum received power level increases as the number of transmitters increases. and the channel gain of one user does not affect the power level of another user as its effects are canceled out by the SIC receiver.

### III. OPTIMUM POWER LEVELS

The optimum received power level, (6), is evaluated using MATLAB for different values of  $N$  and different values of  $\text{SINR}$ . The optimum received power level for different values of  $N$ , where  $\text{SINR}=10$  is fixed for each user, is shown in Fig. 3. The optimal received power levels for different values of  $\text{SINR}$  are depicted in Fig. 4.

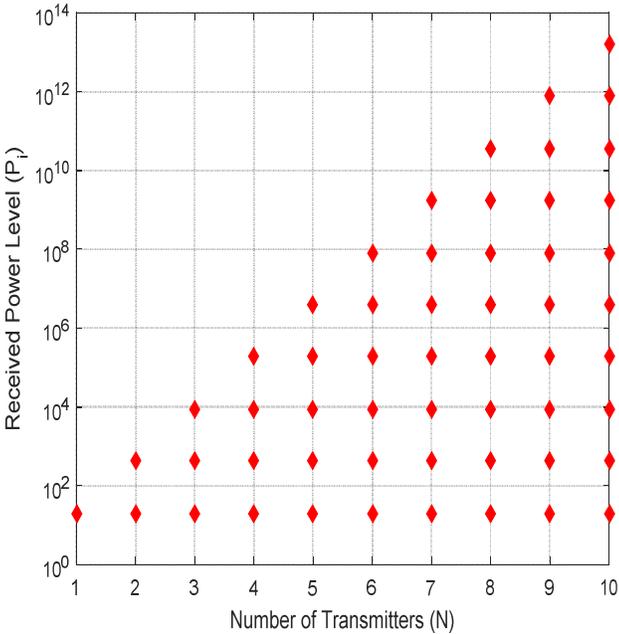


Fig. 3. Received power levels for different number of transmitters  $N$ , where  $\text{SINR} = 10$  dB.

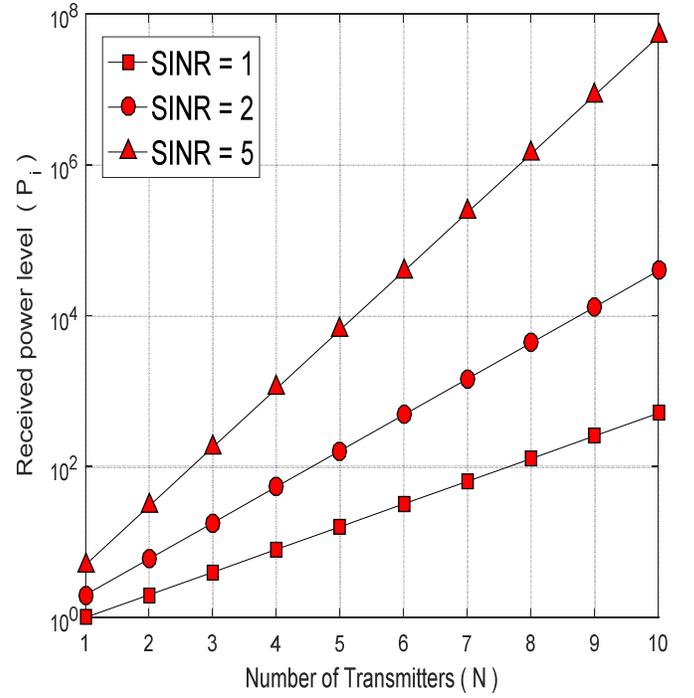


Fig. 4. Received power levels for  $N=10$  and different  $\text{SINR}$  values.

As we show below in Fig. 5, assuming a AWGN channel, the optimum power levels are very similar to the  $\mu$ -law encoding used in PCM speech companders, where the ratio of signal power to quantization noise is kept constant. The  $\mu$ -law compander used in classic telephony PCM is given as [19]

$$F(x) = \text{sgn}(x) \frac{\ln(1 + \mu|x|)}{\ln(1 + \mu)}, \quad (7)$$

where  $x$  is the signal input amplitude and the companding parameter  $\mu$  is equal to 255 in the standard PCM system in the North-America and Japan.

The comparison of optimum received power level with the  $\mu$ -law PCM output levels is shown in Fig. 5. As shown in Fig.5, a  $\mu$ -law compander has a linearly increasing relation in the companded signal and is remarkably similar to the optimum received NOMA power levels shown in Fig 3 and Fig. 4. This similarity is because the design criteria for  $\mu$ -law coding, keeping the ratio of the signal power to quantization noise constant for all signal levels, is very similar to the NOMA requirement of constant received  $\text{SINR}$  for each received signal.

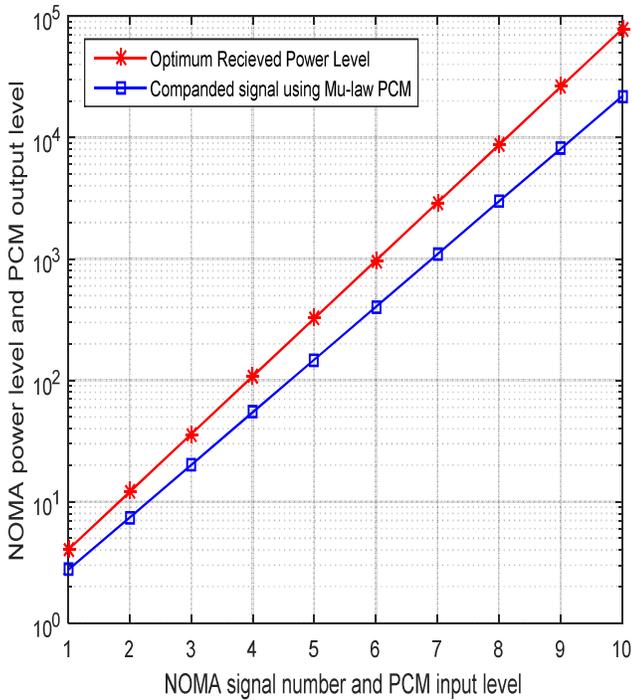


Fig. 5. NOMA optimum power levels versus  $\mu$ -law levels.

#### IV. CONCLUSION

In this paper, a formula for the optimum received power level for uplink power-domain NOMA with ideal SIC reception is derived. The derived results show that the optimum received power level increases linearly (in dB) as the number of transmitters  $N$  are increased and the maximum required received SINR increases exponentially (or equivalently, linearly in dB) with the number of users  $N$ . An interesting observation is that the optimum power levels are very similar to that of the  $\mu$ -law encoding used in the PCM speech companders.

#### ACKNOWLEDGMENT

Faeik Al Rabee is supported by Al-Balqa' Applied University (BAU), Jordan. The statements made herein are solely the responsibility of the authors.

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